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Theoretical study of damping in coupled quantum dots

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Abstract

We present a theoretical study of the damping phenomenon in a device of coupled quantum dots, in the context of quantum computation. We derive the one-phonon contribution to the damping rate, based on Fermi's golden rule, for the case where the device lies in the super-Ohmic regime. The result is in agreement with that of Leggett *et al* derived within the noninteracting blip approximation (NIBA). Moreover, we consider the multi-phonon contribution to damping according to Würger, and estimate the temperature range for the coupled dots to operate as a qubit device. A threshold temperature T^*, 18 K, is determined, above which the tunnelling time becomes longer than the damping time.

1. Introduction

Quantum computation has been an emerging field during the last decade. In this field, people hope to build a new generation computer, a quantum computer, by the principle of quantum mechanics. The basic unit of a quantum computer is called a quantum bit (qubit) expressing states '0' or '1'. Because of the superposition principle, a qubit can take on 0 and 1 states simultaneously.

There are many proposals to construct a qubit device, such as SQUIDs [1], nuclear spins [2], ion traps [3] and quantum dots [4]. An important issue is that a qubit state does not last forever when a qubit is coupled with the environment. This phenomenon is called 'damping,' and damping time (the time it takes for the state to relax) determines whether a qubit state lives long enough for an operation to execute in a pre-set timescale.

The present work focuses on a theoretical study of the damping phenomenon in the device of coupled quantum dots embedded in the oxide layer of a metal–oxide–semiconductor (MOS) proposed by Tanamoto [4]. Each pair of coupled dots forms a qubit, with 0 (1) corresponding to the state where the electron is confined in one (the other) dot, and can be used to implement the fundamental quantum gate, the controlled-not-gate. This proposal has two advantages. Firstly,

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it uses the mature Si technology. This solves the 'scale-up' problem in the case of nuclear spins or ion traps. Secondly, generally speaking, solid state devices allow one to easily read and write qubit information through voltage gates, for example. However, the second advantage is earned only at a price. Because the coupled dots are embedded in SiO_2, there are a large number of background phonons in the vicinity of the electron. These phonons interact with the electron and cause damping of the electron state. Therefore, it is always important to check the damping rate in the case of a solid state qubit device. Tanamoto has estimated the damping rate based on the theoretical expression of Leggett *et al* [5]. The present work tries to improve the estimate in the two following respects. Firstly, we derive explicitly the one-phonon contribution to damping rate, based on the Fermi golden rule. This permits an estimate of the rate at low temperatures. Secondly, we consider the multi-phonon contribution to damping according to Würger [6], and also obtain a high-temperate estimate.

2. Fermi's golden rule approach

2.1. Description of the problem

In insulated materials, acoustic phonons provide the most important damping mechanism. The phonon bath spectrum is defined as

$$J(\omega) \equiv (\pi/2) \sum_q |\lambda_q|^2 \delta(\omega - \omega_q). \tag{1}$$

Equation (1) accounts for various situations in solid-state physics. We work within the Debye model and, after summing over q, the phonon bath spectrum then obeys a cubic law [6]

$$J(\omega) = \pi \alpha \omega^3 \tag{2}$$

where the dimension of the coupling constant α is (frequency)$^{-2}$. It is related to material constants through

$$\alpha = \frac{1}{2\pi^2 \hbar \rho} \left(\frac{\gamma_l^2}{v_l^5} + \frac{2\gamma_t^2}{v_t^5} \right) \equiv \frac{1}{2\pi^2 \hbar \rho} \frac{3\gamma^2}{v^5} \tag{3}$$

where v and γ are appropriate average values of transverse and longitudinal sound velocities, v_t and v_l, and deformation potentials γ_t and γ_l, respectively. ρ is the mass density of the solid. (2) means that the device lies in the so-called super-Ohmic region [5], where $J(\omega) \sim \omega^s$ with $s > 1$. We also take the energy levels of two dots to coincide, so that we may use a simplified physical model, i.e. the symmetric structure of coupled dots, as shown in figure 1. This is appropriate for understanding the switch of the qubit state between 0 and 1.

We write down our Hamiltonian, which describes the coupled quantum dots

$$H = \sum_q \left(a_q^+ a_q + \frac{1}{2} \right) \hbar \omega_q - \frac{\hbar}{2} \Delta_b (C_l^+ C_r + C_r^+ C_l) + \sum_q \frac{\lambda_q}{2} (C_r^+ C_r - C_l^+ C_l)(a_{-q}^+ + a_q) \tag{4}$$

where phonon operators obey bosonic commutation relations $[a_i, a_j^+] = \delta_{ij}$, and electron operators satisfy fermionic commutation relations $\{C_i, C_j^+\} = \delta_{ij}$. Δ_b is the bare electron tunnelling frequency. (Here, 'bare' means that the electron uncouples with phonons.) λ_q is the electron–phonon coupling constant, which is proportional to the inverse of the root of the system volume. (4) describes the well known spin-boson problem [5]. There are various approaches to calculate the damping time of this system, such as NIBA [5, 7], strong-coupling theory [6], and perturbation theory [8]. Although Leggett *et al* [5] demonstrate that the Fermi golden rule is inadequate for the calculation in the case where $J(\omega) \sim \omega^s$ with $s > 2$, we shall

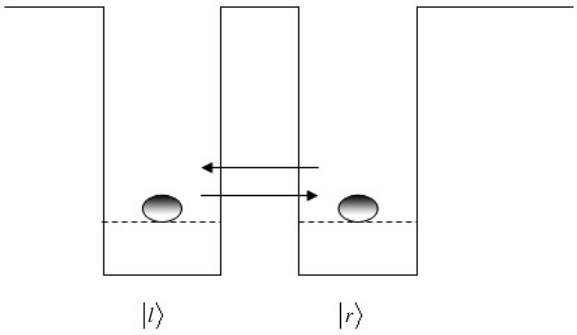

Figure 1. Simplified model of the coupled quantum dots. An electron tunnels back and forth between right and left quantum dots.

show that, switching to different unperturbed eigenfunctions, the damping rate with explicitly only one-phonon contribution can be obtained within the golden rule. Moreover, the result agrees with that of Leggett *et al* [5] derived within the approximation NIBA. Roughly speaking, they calculated the probability $p(t)$ for an electron to go back to the dot where it sits initially, which is given by the following double path integral:

$$p(t) = \int D[x(\tau)] \int D[y(\tau')]A[x(\tau)]A^*[y(\tau')]F[x(\tau), y(\tau')],$$

where $\int D[x(\tau)]A[x(\tau)]$ is the corresponding probability amplitude in the absence of electron–environment interaction, and F is the influence functional accounting for the interaction. $x(\tau)$ (or $y(\tau)$) denotes the position of the electron as a function of time, which, in the case of double dots, can take only two discrete values, e.g. 0 and 1, corresponding to the left and right dots, respectively. They showed that a certain contribution to $p(t)$ involves products of the blip function B

$$B(\tau)B(s) = \sum_{1}^{n} b_j[\Theta(\tau - t_{2j-1}) - \Theta(\tau - t_{2j})] \sum_{1}^{n} b_k[\Theta(s - t_{2k-1}) - \Theta(s - t_{2k})].$$

In such a case they showed that only those with $j = k$ (the same blip) in the product are important, and they made the approximation where the ones with $j \neq k$ were thrown away, hence called the noninteracting blip approximation. The result of Leggett *et al* is not easily accessible to interpretation as consisting of only a one-phonon contribution. We now describe our derivation via a different approach, in the following.

2.2. Break of the Hamiltonian

We show how to break the Hamiltonian into an unperturbed part and the perturbation. Since SiO_2 is a polar material, an electron in SiO_2 will attract a large number of phonons in the vicinity. The polaron concept [9] is used to describe the electron dragging a large number of phonons, and the standard method to tackle this problem is the canonical transformation to localized polaron basis [10]

$$\bar{H} = \mathrm{e}^{S} H \mathrm{e}^{-S}. \tag{5}$$

With

$$S = -\sum_{q} \frac{(\lambda_q/2)}{\omega_q}(C_r^+ C_r - C_1^+ C_1)(a_q - a_{-q}^+) \tag{6}$$

we have

$$\bar{H} = \sum_q \left(a_q^+ a_q + \frac{1}{2}\right)\hbar\omega_q - \sum_q \frac{|\lambda_q/2|^2}{\omega_q} - \frac{\hbar\Delta_b}{2}(C_1^+ C_r X + C_r^+ C_1 X^{-1}) \tag{7}$$

where

$$X = \exp\left[\sum_q \frac{\lambda_q}{\omega_q}(a_q - a_{-q}^+)\right]. \tag{8}$$

Note that $X^\dagger = X^{-1}$ and $\lambda_q^* = \lambda_{-q}$.

Next, we change localized electron operators of (7) to the delocalized operators

$$C_s^+ = (C_1^+ + C_r^+)/\sqrt{2} \tag{9}$$

$$C_a^+ = (C_1^+ - C_r^+)/\sqrt{2} \tag{10}$$

corresponding to symmetric and anti-symmetric states, which gives

$$\bar{H} = \sum_q \left(a_q^+ a_q + \frac{1}{2}\right)\hbar\omega_q - \sum_q \left(\frac{|\lambda_q/2|^2}{\omega_q}\right) - \frac{\hbar\Delta_b}{4}(C_s^+ C_s - C_a^+ C_a)(X + X^{-1}) - \frac{\hbar\Delta_b}{4}(C_a^+ C_s - C_s^+ C_a)(X - X^{-1}). \tag{11}$$

To break the Hamiltonian, we re-write the operator in the last term by inserting a complete set $|i, n_q\rangle; i = s, a$ twice, i.e. $A = \sum_{a''}\sum_{a'} |a''\rangle\langle a''|A|a'\rangle\langle a'|$. Then, we separate diagonal and nondiagonal parts of the matrix. After these two steps, (11) becomes

$$\bar{H} = H_0 + V_1 + V_2. \tag{12}$$

The first term is the unperturbed Hamiltonian

$$H_0 = \sum_q \left(a_q^+ a_q + \frac{1}{2}\right)\hbar\omega_q - \sum_q \frac{|\lambda_q/2|^2}{\omega_q} - \frac{\hbar\Delta_b}{4}\sum_{n_q}\langle n_q|X + X^{-1}|n_q\rangle(|s, n_q\rangle\langle s, n_q| - |a, n_q\rangle\langle a, n_q|) \tag{13}$$

and the last two terms together represent the perturbation

$$V_1 = -\frac{\hbar\Delta_b}{4}(C_a^+ C_s - C_s^+ C_a)(X - X^{-1}) \tag{14}$$

$$V_2 = -\frac{\hbar\Delta_b}{4}\sum_{\substack{n_q, n_q' \\ n_q \neq n_q'}} \langle n_q'|X + X^{-1}|n_q\rangle(|s, n_q'\rangle\langle s, n_q| - |a, n_q'\rangle\langle a, n_q|). \tag{15}$$

Thus, our unperturbed eigenstates are the symmetric and anti-symmetric polaron states plus free phonons.

Note that Leggett *et al* [5] show that, if localized polarons are used as unperturbed states, the golden rule does not yield the correct damping rate in the case $J(\omega) \sim \omega^s$ with $s > 2$. However, in contrast, we have changed the localized states to extended (symmetric and anti-symmetric) ones, and (12) describes the situation where the extended electron state, due to its interaction with the environment (as represented by the perturbation), will lose phase coherence.

2.3. One-phonon damping rate

At a low temperature where there are only a few phonons in the system, the one-phonon process dominates the transition between symmetric and antisymmetric states, with the phonon occupation number only changing by one during the transition. In the following, we show how to derive the one-phonon contribution to the damping rate. Firstly, note that since

$$V_1 \propto (C_a^+ C_s - C_s^+ C_a)(X - X^{-1}) \propto (C_a^+ C_s - C_s^+ C_a)\left(\sum_q (\lambda_q/\omega_q)(a_q - a_{-q}^+)\right)^n$$

where $n =$ odd, V_1 contributes to the one-phonon transition. On the other hand,

$$V_2 \propto \sum \langle n_q'|X + X^{-1}|n_q\rangle(|s, n_q'\rangle\langle s, n_q| - |a, n_q'\rangle\langle a, n_q|)$$
$$\propto \sum \langle n_q'|\left(\sum_q (\lambda_q/\omega_q)(a_q - a_{-q}^+)\right)^n |n_q\rangle(|s, n_q'\rangle\langle s, n_q| - |a, n_q'\rangle\langle a, n_q|)$$

where $n =$ even, and so V_2 does not enter the calculation of the one-phonon damping rate.

Fermi's golden rule gives the damping rate of the state i as

$$\Gamma_i = (2\pi/\hbar)\sum_{f\neq i} |V_{fi}|^2 \delta(E_f - E_i) \tag{16}$$

whose inverse is the damping time

$$\tau_i = \frac{1}{\Gamma_i}. \tag{17}$$

According to (13), our initial and final unperturbed states and energy are, respectively,

$$\left|s, n_q^i\right\rangle, \qquad E_{s,n_q^i} = \hbar\left(\sum_q \left(n_q^i + \frac{1}{2}\right)\omega_q - \frac{\Delta_b}{4}\langle n_q^i|X + X^{-1}|n_q^i\rangle\right) - \sum_q \frac{|\lambda_q/2|^2}{\omega_q} \tag{18}$$

$$\left|a, n_q^f\right\rangle, \qquad E_{a,n_q^f} = \hbar\left(\sum_q \left(n_q^f + \frac{1}{2}\right)\omega_q + \frac{\Delta_b}{4}\langle n_q^f|X + X^{-1}|n_q^f\rangle\right) - \sum_q \frac{|\lambda_q/2|^2}{\omega_q}. \tag{19}$$

Substituting equations (14), (18), and (19) into (16), we obtain the damping rate of the symmetric state, due to its transition to the anti-symmetric state,

$$\Gamma_s^{(\mathrm{odd})} = 2\pi \sum_{n_q^f}\left[\left(\frac{\Delta_b}{4}\right)^2 |\langle n_q^f|X - X^{-1}|n_q^i\rangle|^2 \right.$$
$$\times \delta\left(\sum_q n_q^f\omega_q - \sum_q n_q^i\omega_q + \frac{\Delta_b}{4}\langle n_q^f|X + X^{-1}|n_q^f\rangle\right.$$
$$\left.\left. + \frac{\Delta_b}{4}\langle n_q^i|X + X^{-1}|n_q^i\rangle\right)\right] \tag{20}$$

that of the antisymmetric state can be similarly calculated. Using the identity $\int_{-\infty}^{\infty} \mathrm{d}t \exp(\mathrm{i}ft) = 2\pi\delta(f)$ in (20), we obtain

$$\Gamma_s^{(\mathrm{odd})} = \int_{-\infty}^{\infty} \mathrm{d}t \sum_{n_q^f}\left(\frac{\Delta_b}{4}\right)^2 \mathrm{e}^{\mathrm{i}\frac{\Delta_b}{4}[\langle n_q^f|X+X^{-1}|n_q^f\rangle + \langle n_q^i|X+X^{-1}|n_q^i\rangle]t}$$
$$\times \langle n_q^i|(X^{-1} - X)|n_q^f\rangle\langle n_q^f|(X(t) - X^{-1}(t))|n_q^i\rangle \tag{21}$$

where

$$X(t) = \exp\left[\sum_q \frac{\lambda_q}{\omega_q}(\mathrm{e}^{-\mathrm{i}\omega_q t}a_q - \mathrm{e}^{\mathrm{i}\omega_q t}a_{-q}^+)\right]. \tag{22}$$

Now, we extract, from (21), the one-phonon process

$$\Gamma_s^{(1)} = \text{term1} + \text{term2} \tag{23}$$

where

$$\text{term1} = \int_{-\infty}^{\infty} \mathrm{d}t \sum_q \left(\frac{\Delta_\mathrm{b}}{4}\right)^2 \mathrm{e}^{\mathrm{i}\frac{\Delta_\mathrm{b}}{4}[\langle n-1|X+X^{-1}|n-1\rangle + \langle n|X+X^{-1}|n\rangle]t} \times \langle n|(X^{-1}-X)|n-1\rangle\langle n-1|(X(t)-X^{-1}(t))|n\rangle \tag{24}$$

and

$$\text{term2} = \int_{-\infty}^{\infty} \mathrm{d}t \sum_q \left(\frac{\Delta_\mathrm{b}}{4}\right)^2 \mathrm{e}^{\mathrm{i}\frac{\Delta_\mathrm{b}}{4}[\langle n+1|X+X^{-1}|n+1\rangle + \langle n|X+X^{-1}|n\rangle]t} \times \langle n|(X^{-1}-X)|n+1\rangle\langle n+1|(X(t)-X^{-1}(t))|n\rangle \tag{25}$$

with the symbol n replacing n_q^i for the sake of abbreviation. Next, we discuss how to calculate (24), and (25) can be done in a similar way. In (24), the matrix elements are evaluated as below

$$\langle n-1|X+X^{-1}|n-1\rangle = 2\mathrm{e}^{-|u_q|^2/2}[L_{n_q^i-1}(|u_q|^2)]\left[\prod_{q'\neq q} \mathrm{e}^{-|u_{q'}|^2/2} L_{n_{q'}^i}(|u_{q'}|^2)\right] \tag{26}$$

$$\langle n|X+X^{-1}|n\rangle = 2\prod_{q'} \mathrm{e}^{-|u_{q'}|^2/2} L_{n_{q'}^i}(|u_{q'}|^2) \tag{27}$$

$$\langle n|X^{-1}-X|n-1\rangle\langle n-1|X(t)-X^{-1}(t)|n\rangle = 4\left\{(\mathrm{e}^{-\mathrm{i}\omega_q t}\mathrm{e}^{-|u_q|^2}|u_q|^2)\left(\frac{1}{n_q^i}\right) \times \left[L_{n_q^i-1}^{(1)}(|u_q|^2)\right]^2\right\}\left\{\prod_{q'\neq q}[\mathrm{e}^{-|u_{q'}|^2/2} L_{n_{q'}^i}(|u_{q'}|^2)]^2\right\} \tag{28}$$

where $u_q \equiv \lambda_q/\omega_q \propto \lambda_q \propto 1/\sqrt{V}$, V is the system volume, L_{n-1} and L_n are Laguerre polynomials of order $n-1$ and n, respectively, and $L_{n-1}^{(1)}$ is an associated Laguerre polynomial. Note that $|u_q|^2 \propto 1/V$, and so in the thermodynamic limit we only need to retain the linear term of $|u_q|^2$. In (24), expanding the exponential term, substituting the expressions from equations (26)–(28), and keeping up to the linear terms of $|u_q|^2$, it becomes

$$\begin{aligned}\text{term1} = \int_{-\infty}^{\infty} \mathrm{d}t \left(\frac{\Delta_\mathrm{b}}{4}\right)^2 \sum_q \Bigg\{ & 4(\mathrm{e}^{-\mathrm{i}\omega_q t}|u_q|^2) n_q^i \left[\prod_{q'} \mathrm{e}^{-|u_{q'}|^2/2} L_{n_{q'}^i}(|u_{q'}|^2)\right]^2 \\ & + 4(\mathrm{e}^{-\mathrm{i}\omega_q t}|u_q|^2) n_q^i \left[\prod_{q'} \mathrm{e}^{-|u_{q'}|^2/2} L_{n_{q'}^i}(|u_{q'}|^2)\right]^3 (\mathrm{i}\Delta_\mathrm{b} t) \\ & + 4(\mathrm{e}^{-\mathrm{i}\omega_q t}|u_q|^2) n_q^i \left[\prod_{q'} \mathrm{e}^{-|u_{q'}|^2/2} L_{n_{q'}^i}(|u_{q'}|^2)\right]^4 \frac{(\mathrm{i}\Delta_\mathrm{b} t)^2}{2} + \cdots \Bigg\}.\end{aligned} \tag{29}$$

Next, we take the thermal average of (29) with respect to the phonon bath as follows. Note firstly that, in the thermodynamic limit, the linear term of $|u_{q'}|^2$ in $L_{n_{q'}^i}$ gives

$$\left\langle \left[\prod_{q'} \mathrm{e}^{-|u_{q'}|^2/2} L_{n_{q'}^i}(|u_{q'}|^2)\right]^x \right\rangle_e = \left\langle \prod_{q'} \mathrm{e}^{-x|u_{q'}|^2/2}(1-x|u_{q'}|^2 n_{q'}^i) \right\rangle_e = \prod_{q'} \mathrm{e}^{-x|u_{q'}|^2/2}(1-x|u_{q'}|^2 N_{q'})$$

where $\langle A\rangle_e = \mathrm{tr_B}(A\mathrm{e}^{-\beta a_q^+ a_q \omega_q})/Z$ denotes the thermal average, $Z = \prod_q 1/(1-\mathrm{e}^{-\beta\omega_q})$, $N_q = \langle n_q\rangle_e = 1/(\mathrm{e}^{\beta\hbar\omega_q}-1)$ and $x = 2, 3, 4, \ldots$. Moreover, making the customary exponential approximation, the last expression becomes

$$\left\langle\left[\prod_{q'}\mathrm{e}^{-|u_{q'}|^2/2}L_{n^i_{q'}}(|u_{q'}|^2)\right]^x\right\rangle_e \approx \left[\prod_{q'}\mathrm{e}^{-|u_{q'}|^2/2}\mathrm{e}^{-|u_{q'}|^2 N_{q'}}\right]^x$$
$$= \left[\prod_{q'}\mathrm{e}^{\frac{-|u_{q'}|^2}{2}(2N_{q'}+1)}\right]^x. \tag{30}$$

Now, inserting the above result in the thermal average of (29), it gives

$$\langle\mathrm{term1}\rangle_e \approx \int_{-\infty}^{\infty}\mathrm{d}t\left(\frac{\Delta_\mathrm{b}}{4}\right)^2\sum_q\left\{4(\mathrm{e}^{-\mathrm{i}\omega_q t}|u_q|^2)N_q\left[\prod_{q'}\mathrm{e}^{\frac{-|u_{q'}|^2}{2}(2N_{q'}+1)}\right]^2\right.$$
$$+ 4(\mathrm{e}^{-\mathrm{i}\omega_q t}|u_q|^2)N_q\left[\prod_{q'}\mathrm{e}^{\frac{-|u_{q'}|^2}{2}(2N_{q'}+1)}\right]^3(\mathrm{i}\Delta_\mathrm{b}t)$$
$$\left.+ 4(\mathrm{e}^{-\mathrm{i}\omega_q t}|u_q|^2)N_q\left[\prod_{q'}\mathrm{e}^{\frac{-|u_{q'}|^2}{2}(2N_{q'}+1)}\right]^4\frac{(\mathrm{i}\Delta_\mathrm{b}t)^2}{2}+\cdots\cdots\right\}. \tag{31}$$

The above expression can be rewritten in a condensed form as follows. We introduce

$$\tilde{\Delta} \equiv \Delta_\mathrm{b}\mathrm{e}^{-S_T} \tag{32}$$

where

$$\mathrm{e}^{-S_T} = \prod_{q'}\mathrm{e}^{\frac{-|u_{q'}|^2}{2}(2N_{q'}+1)} = \exp\left[\sum_{q'}\frac{-|u_{q'}|^2}{2}\coth\left(\frac{\hbar\omega_q}{k_\mathrm{B}T}\right)\right] \tag{33}$$

(32) is the reduced tunnelling frequency [5, 6] and (33) is the Debye–Waller factor [10]. Arranging (31) in the form of (32), we obtain

$$\langle\mathrm{term1}\rangle_e \approx \int_{-\infty}^{\infty}\mathrm{d}t\left(\frac{\tilde{\Delta}}{2}\right)^2\sum_q|u_q|^2N_q\mathrm{e}^{-\mathrm{i}\omega_q t}\mathrm{e}^{\mathrm{i}\tilde{\Delta}t}. \tag{34}$$

Similarly, the thermal average of term 2 of (23) is

$$\langle\mathrm{term2}\rangle_e \approx \int_{-\infty}^{\infty}\mathrm{d}t\left(\frac{\tilde{\Delta}}{2}\right)^2\sum_q|u_q|^2(N_q+1)\mathrm{e}^{\mathrm{i}\omega_q t}\mathrm{e}^{\mathrm{i}\tilde{\Delta}t}. \tag{35}$$

With equations (34) and (35), (23) becomes

$$\langle\Gamma_s^{(1)}\rangle_e = \langle\mathrm{term1}\rangle_e + \langle\mathrm{term2}\rangle_e \approx \int_{-\infty}^{\infty}\mathrm{d}t\left(\frac{\tilde{\Delta}}{2}\right)^2\sum_q[|u_q|^2N_q\mathrm{e}^{\mathrm{i}(\tilde{\Delta}-\omega_q)t}$$
$$+ |u_q|^2(N_q+1)\mathrm{e}^{\mathrm{i}(\tilde{\Delta}+\omega_q)t}]. \tag{36}$$

After integrating (36) with respect to time, we obtain the one-phonon damping rate of the symmetric state

$$\langle\Gamma_s^{(1)}\rangle_e \approx \left(\frac{\tilde{\Delta}}{2}\right)^2 2\pi\sum_q[|u_q|^2N_q\delta(\tilde{\Delta}-\omega_q) + |u_q|^2(N_q+1)\delta(\tilde{\Delta}+\omega_q)]. \tag{37}$$

Following the same steps as above, that of the anti-symmetric state is

$$\langle\Gamma_a^{(1)}\rangle_e \approx \left(\frac{\tilde{\Delta}}{2}\right)^2 2\pi\sum_q[|u_q|^2N_q\delta(\tilde{\Delta}+\omega_q) + |u_q|^2(N_q+1)\delta(\tilde{\Delta}-\omega_q)]. \tag{38}$$

Defining the average one-phonon damping rate as

$$\langle\Gamma^{(1)}\rangle_e \equiv \frac{\langle\Gamma_s^{(1)}\rangle_e + \langle\Gamma_a^{(1)}\rangle_e}{2} \tag{39}$$

we obtain

$$\langle\Gamma^{(1)}\rangle_e \approx \left(\frac{\tilde{\Delta}}{2}\right)^2 \pi \sum_q \left[|u_q|^2(\delta(\tilde{\Delta}+\omega_q)+\delta(\tilde{\Delta}-\omega_q))\coth\left(\frac{\hbar\omega_q}{2k_\mathrm{B}T}\right)\right] \tag{40}$$

where we have used $2N_q + 1 = \coth(\beta\hbar\omega_q/2)$. Because of energy conservation, the first delta function in the bracket has no contribution, and so

$$\langle\Gamma^{(1)}\rangle_e \approx \left(\frac{\tilde{\Delta}}{2}\right)^2 \pi \sum_q \left[\frac{|\lambda_q|^2}{\omega_q^2}\delta(\tilde{\Delta}-\omega_q)\coth\left(\frac{\hbar\omega_q}{2k_\mathrm{B}T}\right)\right]. \tag{41}$$

In terms of the spectrum function defined in (1), we obtain, finally,

$$\langle\Gamma^{(1)}\rangle_e \approx \frac{1}{2}J(\tilde{\Delta})\coth\left(\frac{\hbar\tilde{\Delta}}{2k_\mathrm{B}T}\right). \tag{42}$$

Note that (42) is identical with the result derived by Leggett *et al* [5] in the case where $J(\omega) \sim \omega^s$ with $s > 2$, within the NIBA. The advantage of our approach is that, unlike the NIBA, the physical picture is clear. We see that the damping mechanism as described in (42) is entirely due to the one-phonon-induced transition between the symmetric and antisymmetric states. Also note that, because the multi-phonon transition becomes important with increasing temperature [6–8], the damping rate derived in this section is actually underestimated in the high temperature range.

3. Estimate of damping time

Firstly, we introduce the following useful mathematical result [6, 10] for the reduced tunnelling frequency:

$$\tilde{\Delta} = \Delta_0 \exp(-\varphi_0/6) \tag{43}$$

where

$$\Delta_0 \equiv \Delta_\mathrm{b}\exp(-\alpha\omega_D^2/2) \tag{44}$$

$$\varphi_0 \equiv \frac{T^2}{T_0^2} = 2\alpha\left(\frac{\pi}{\beta\hbar}\right)^2 \tag{45}$$

$$T_0 \equiv \hbar(\pi k_\mathrm{B}\sqrt{2\alpha}). \tag{46}$$

The above result is obtained by carrying out the wavevector summation in (33) and putting it back in (32), expressing the reduced tunnelling frequency in terms of physical parameters of the system. It will be needed in the following numerical estimation.

3.1. Zero temperature

From equations (42) and (2), the damping rate at zero temperature is

$$\langle\Gamma^{(1)}\rangle_e \approx \tfrac{1}{2}\pi\alpha\tilde{\Delta}^3 \tag{47}$$

where $\tilde{\Delta} = \Delta_0$. With the material parameters quoted in table 1 for SiO_2, the damping time at zero temperature is $\tau = 1/\langle\Gamma^{(1)}\rangle_e \sim 6.5\times10^{-5}$ (s), and the tunnelling time at zero temperature

Table 1. Parameters for SiO_2 [6].

ρ = Mass density = 2200 (kg m^{-3})
Δ_0 = Reduced tunnelling frequency (at $T = 0$) $\approx 4.58 \times 10^9$ (s^{-1})
Θ = Debye temperature $\approx$ 450 (K)
ω_D = Debye frequency $\equiv \frac{k_B\Theta}{\hbar} \approx 5.89 \times 10^{13}$ (s^{-1})
$v_{l,t}$ = Longitudinal/transverse sound velocity $\approx$ 5800/3800 (m s^{-1})
γ = Average deformation potential $\approx$ 2.6 (eV)
From the above parameters, we obtain
(i) Coupling constant $\alpha = 1.01 \times 10^{-25}$ (s^2) according to (3).
(ii) Temperature scale $T_0 = 5.4$ (K) according to (46).

is $\Delta_0^{-1} \sim 2.2 \times 10^{-10}$ (s). This means that the qubit device can switch between 0 and 1 about 10^5 times before appreciable damping occurs.

The damping time estimated above is longer than that of Tanamoto [4], 4.8×10^{-7} (s), for the following reason. Firstly, the parameters of [6] are used in the present work, which are different from Tanamoto's. Secondly, Tanamoto uses the bare frequency Δ_b in place of the reduced frequency Δ_0, for lack of available data, which overestimates Δ_0 and hence the damping rate.

3.2. Damping time versus temperature

In this section, we discuss the damping time versus temperature, based on Würger's work [6]. At low temperatures where $T \ll T_0$ (with T_0 the temperature scale defined in (46)), the damping rate is [6]

$$\langle\Gamma_t\rangle_e \approx \tfrac{1}{2}\pi\alpha(\tilde{\Delta})^3 \coth(\beta\hbar\tilde{\Delta}/2) + \mathrm{O}(\alpha^2). \tag{48}$$

We see that (48) is identical to (42) for the cubic spectrum function, $J(\omega) = \pi\alpha\omega^3$. However, when $T \geqslant T_0$, the multi-phonon damping rate becomes important and the corresponding damping rate is [6]

$$\langle\Gamma_t\rangle_e = \tfrac{1}{2}\gamma_u + \gamma_g \tag{49}$$

where

$$\gamma_u = \frac{1}{2}\tilde{\Delta}^2 \cosh(\beta\hbar\tilde{\Delta}/2)\left(\frac{\beta\hbar}{\pi}\right)\sum_{n=\mathrm{odd}}^{\infty}\frac{\varphi_0^n}{n!}A_n(\beta\hbar\tilde{\Delta}) \tag{49a}$$

$$\gamma_g = \frac{1}{2}\tilde{\Delta}^2\left(\frac{\beta\hbar}{\pi}\right)\sum_{n=\mathrm{even}}^{\infty}\frac{\varphi_0^n}{n!}A_n(0) \tag{49b}$$

with

$$A_n(\beta\hbar\omega) = \frac{\pi}{\beta\hbar}\int_{-\infty}^{\infty}\mathrm{d}t\,\mathrm{e}^{\mathrm{i}\omega t}\frac{1}{\cosh(\pi t/\beta\hbar)^{2n}}. \tag{49c}$$

Because (49) has no closed form, Würger proposed an interpolation method to approximate (49) [6],

$$\langle\Gamma_t\rangle_e = \tfrac{1}{2}\gamma_u + \gamma_g \tag{50}$$

$$\gamma_u = \pi\alpha\tilde{\Delta}^3 \coth\left(\frac{1}{2}\beta\hbar\tilde{\Delta}\right)\frac{\sinh(\varphi_0)}{\varphi_0\sqrt{1+4\varphi_0/\pi}} \tag{50a}$$

$$\gamma_g = 2\pi\alpha\tilde{\Delta}^2(k_BT/\hbar)\frac{\cosh(\varphi_0)-1}{\varphi_0\sqrt{1+4\varphi_0/\pi}}. \tag{50b}$$

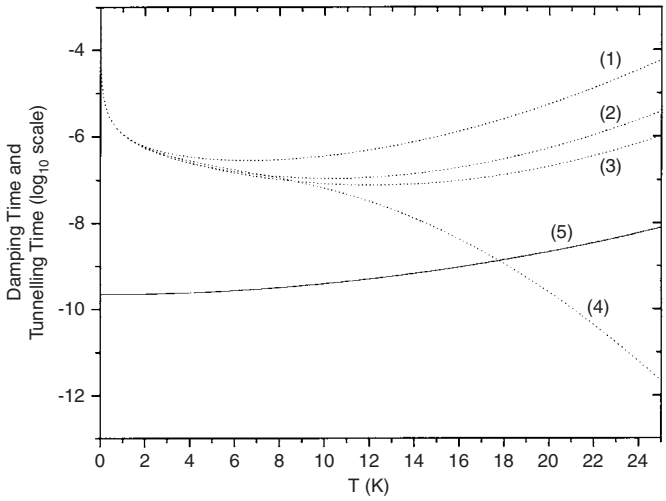

Figure 2. Damping time and tunnelling time versus temperature. Curve 1: one-phonon damping time from (48). Curve 2: the damping time with contributions from the first two terms of (49*a*) and (49*b*). Curve 3: the damping time with contributions from the first three terms of (49*a*) and (49*b*). Curve 4: the damping time according to the approximation (50). Curve 5: tunnelling time from (43).

We plot the relation of damping time and reduced tunnelling time to temperature in figure 2. From curves 1, 2, and 3, we find that as we consider more terms in (49), the result moves downward to curve 4, which is the approximation of (49). Curve 5, the tunnelling time, increases with the temperature, by about a factor of ten in the temperature range from $T = 0$ to 25 K. If we regard the inverse tunnelling time as the switching speed of a quantum gate, then it means that the speed of a CPU made of such quantum gates is sensitive to temperature variation. On the other hand, curve 4 shows that the damping time decreases rapidly as T increases, and there is an intersection between curves 4 and 5. At that point the damping time is equal to the tunnelling time, and the qubit cannot complete a switching cycle between 0 and 1 before the damping occurs. According to figure 2, the corresponding temperature is about 18 K, and we denote this threshold temperature T^*. Therefore, at $T \geqslant T^*$, coherent tunnelling is destroyed.

4. Conclusions

We present an approach based on Fermi's golden rule to derive the one-phonon damping rate of coupled quantum dots, in the case $J(\omega) \sim \omega^s$ with $s > 2$, which is found to be in agreement with that of Leggett *et al* derived within the NIBA [5]. The damping time at zero temperature is estimated to be 6.5×10^{-5} (s), longer than Tanamoto's estimate [4]. As temperature increases, the multi-phonon effect becomes important, and a numerical estimate of the rate is carried out based on the result of Würger [6]. Overall, according to the present work, double quantum dots show interesting properties not far off the target for qubit application, but it also points out some issues for future research. Firstly, there is a threshold temperature T^*, i.e. 18 K, above which the tunnelling time becomes longer than the damping time, which sets an upper temperature bound for the operation of the coupled quantum dots as a qubit device. Secondly, the switching speed of a quantum gate made of such double quantum dots is temperature sensitive, and reduces substantially as the temperature is raised. Future study to optimize the structure towards a higher T^* and a less thermal sensitivity may be worthwhile.

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